

KEY CONSTANTS

- $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m}\cdot\text{A}^{-1}$ (Permeability of free space)
- $\frac{\mu_0}{4\pi} = 10^{-7} \text{ T}\cdot\text{m}\cdot\text{A}^{-1}$
- Charge of electron: $e = 1.6 \times 10^{-19} \text{ C}$
- $\frac{\mu_0}{4\pi} \approx 10^{-7}$ (Use in Biot-Savart problems)

1. MAGNETIC FORCE ON A MOVING CHARGE (LORENTZ FORCE)

1.1 Lorentz Force

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$F = qvB \sin \theta$$

Symbols:

- q = charge (C); v = speed (m/s)
- B = magnetic field ($\text{T} = \text{kg}\cdot\text{A}^{-1}\cdot\text{s}^{-2}$)
- θ = angle between \vec{v} and \vec{B}

Conditions:

- $F = 0$ when $\theta = 0$ or 180 (parallel/anti-parallel)
- $F = qvB$ (maximum) when $\theta = 90$
- Direction: Use **Fleming's Left Hand Rule** or right-hand rule for $\vec{v} \times \vec{B}$ (then apply sign of q)

Common Mistake: Magnetic force does *no work* because $\vec{F} \perp \vec{v}$ always. So KE and speed remain constant.

1.2 Combined Lorentz Force (Electric + Magnetic)

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

1.3 Velocity Selector (Filter)

$$v = \frac{E}{B}$$

- Condition: $qE = qvB \Rightarrow v = E/B$
- Used in mass spectrometers to select a specific speed
- Independent of charge and mass of the particle

Tip: In velocity selector, only particles with $v = E/B$ pass undeflected regardless of their charge-to-mass ratio.

1.4 Motion of Charged Particle in Magnetic Field

Radius of circular motion:

$$r = \frac{mv}{qB}$$

m = mass (kg), v = speed (m/s)

Also: $r = \frac{p}{qB}$ ($p = mv$ = momentum)

Also: $r = \frac{\sqrt{2mK}}{qB}$ (K = KE in J)

Also: $r = \frac{\sqrt{2mqV}}{qB} = \frac{\sqrt{2mV/q}}{B}$ (V = accelerating voltage)

Time period & frequency:

$$T = \frac{2\pi m}{qB}$$

$$f = \frac{qB}{2\pi m}$$

$$\omega = \frac{qB}{m}$$

- T is independent of v and r
- This is the principle of **Cyclotron**

Pitch of helical motion:

$$p = v_{\parallel} \cdot T = v \cos \theta \cdot \frac{2\pi m}{qB}$$

- When \vec{v} is at angle θ to \vec{B}
- $v_{\perp} = v \sin \theta \rightarrow$ circular motion
- $v_{\parallel} = v \cos \theta \rightarrow$ uniform linear motion
- Combined: **Helical path**

Confused Formula Alert: $r \propto \sqrt{m}$ for same KE but $r \propto m$ for same speed. Use correct formula: $r = \sqrt{2mK}/(qB)$ for KE and $r = mv/(qB)$ for speed.

2. MAGNETIC FORCE ON A CURRENT-CARRYING CONDUCTOR

2.1 Force on a Straight Conductor

$$\vec{F} = I(\vec{L} \times \vec{B})$$

$$F = BIL \sin \theta$$

- θ = angle between current direction and \vec{B}
- $F = 0$ when conductor is parallel to \vec{B}
- $F = BIL$ (max) when conductor $\perp \vec{B}$
- Direction: **Fleming's Left Hand Rule**

- I = current (A), L = length of conductor (m)

2.2 Force Between Two Parallel Wires

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

- d = separation between wires (m)
- Same direction currents** → Attractive force

- Opposite direction currents** → Repulsive force
- Definition of **1 Ampere**: Force = 2×10^{-7} N/m when $I_1 = I_2 = 1$ A, $d = 1$ m

Attraction/repulsion is exactly opposite to that of **electric currents** in wires vs. **static charges** in lines. Two like charges repel; two same-direction currents *attract*.

3. BIOT-SAVART LAW & MAGNETIC FIELD DUE TO CURRENT DISTRIBUTIONS

3.1 Biot-Savart Law (Fundamental)

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I d\vec{l} \times \hat{r}}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I dl \sin \phi}{r^2}$$

- dl = small current element (m)
- r = distance from element to point (m)
- ϕ = angle between $d\vec{l}$ and \hat{r}
- B is in Tesla (T)

3.2 Magnetic Field due to a Moving Point Charge

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{q \vec{v} \times \hat{r}}{r^2}$$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{qv \sin \theta}{r^2}$$

- q = charge (C), v = speed (m/s)
- θ = angle between \vec{v} and \hat{r}
- Analogous to Biot-Savart (replace $I dl \rightarrow qv$)

3.3 Magnetic Field — Standard Configurations

(a) Infinite Straight Wire:

$$B = \frac{\mu_0 I}{2\pi r}$$

- r = perpendicular distance from wire (m)
- $B \propto 1/r$ (hyperbolic variation)
- Field lines are concentric circles around wire

(b) Semi-infinite Wire (at end):

$$B = \frac{\mu_0 I}{4\pi r}$$

- Exactly half of infinite wire result

(c) Finite Wire (at perpendicular bisector):

$$B = \frac{\mu_0 I}{4\pi r} \cdot \frac{2L}{\sqrt{r^2 + L^2}}$$

Or using angles: $B = \frac{\mu_0 I}{4\pi r} (\sin \alpha + \sin \beta)$

- α, β = angles subtended at the point
- For infinite wire: $\alpha = \beta = 90 \Rightarrow B = \mu_0 I / (2\pi r)$

(d) Circular Loop — Centre:

$$B = \frac{\mu_0 I}{2R} \quad (\text{single loop})$$

$$B = \frac{\mu_0 NI}{2R} \quad (N \text{ turns})$$

- R = radius of loop (m)
- Direction: Right-hand thumb rule (curl fingers in current direction, thumb → \vec{B} direction)

(e) Circular Loop — Axial Point:

$$B = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}}$$

- x = axial distance from centre (m)
- At centre ($x = 0$): reduces to $B = \mu_0 I / 2R$
- At $x \gg R$: $B \approx \frac{\mu_0 IR^2}{2x^3}$ (dipole behaviour)

(f) Arc of Circle (angle θ at centre):

$$B = \frac{\mu_0 I \theta}{4\pi R}$$

Semicircle ($\theta = \pi$): $B = \frac{\mu_0 I}{4R}$

3.4 Magnetic Field due to Charge Distributions (Current Densities)

(g) Solid Infinite Cylindrical Wire (radius R):

$$\text{Outside } (r \geq R): \quad B = \frac{\mu_0 I}{2\pi r}$$

$$\text{Inside } (r < R): \quad B = \frac{\mu_0 I r}{2\pi R^2}$$

- Inside: $B \propto r$ (linear increase)
- Outside: $B \propto 1/r$ (hyperbolic decrease)

- At surface ($r = R$): both give $B = \mu_0 I / (2\pi R)$

(h) Hollow Cylindrical Wire (thick shell, inner a , outer b):

$$r < a: B = 0$$

$$a \leq r \leq b: B = \frac{\mu_0 I}{2\pi r} \cdot \frac{r^2 - a^2}{b^2 - a^2}$$

$$r > b: B = \frac{\mu_0 I}{2\pi r}$$

(i) Infinite Plane Sheet of Current (surface density K):

$$B = \frac{\mu_0 K}{2} \quad (\text{on each side, parallel to sheet})$$

- K = surface current density (A/m)
- Field is uniform and parallel to the sheet

Confused Formula Alert: $B_{\text{inside solid wire}} = \frac{\mu_0 I r}{2\pi R^2}$ vs. $B_{\text{inside hollow wire}} = 0$. The hollow wire behaves like a shell — **no field inside the cavity.**

4. AMPERE'S CIRCUITAL LAW

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

- I_{enc} = total current enclosed by the Amperian loop
- Choose circular Amperian loops for symmetric cases
- Analogous to Gauss's Law (electric) but for magnetism

Applications:

- Infinite wire: $B(2\pi r) = \mu_0 I$
- Solenoid: $B \cdot l = \mu_0 n l I$
- Toroid: $B(2\pi r) = \mu_0 N I$

4.2 Toroid

$$B = \frac{\mu_0 N I}{2\pi r} \quad (\text{inside core})$$

$$B = 0 \quad (\text{outside \& in the hole})$$

- N = total turns, r = radius of toroidal loop
- Can write as $B = \mu_0 n I$ where $n = N/(2\pi r)$

4.1 Solenoid

$$B = \mu_0 n I$$

$$n = N/L \quad (\text{turns per unit length})$$

- N = total turns, L = length of solenoid (m)
- Valid for *infinite* (or long) solenoid
- At the **end** of solenoid: $B = \frac{\mu_0 n I}{2}$
- Field is uniform & parallel inside; zero outside

$B = \mu_0 n I$ not $\mu_0 N I$. Always use n (turns/m), not total turns N .

Solenoid vs Toroid:

Solenoid: $B = \mu_0 n I$ (uniform, inside only)
 Toroid: $B = \mu_0 N I / (2\pi r)$ (varies with r , inside core only)
 Both give $B = 0$ in regions without enclosed current.

5. TORQUE ON A CURRENT LOOP & MAGNETIC DIPOLE

5.1 Torque on a Rectangular Loop

$$\vec{\tau} = \vec{m} \times \vec{B}$$

$$\tau = N I A B \sin \theta$$

- N = number of turns
- I = current (A), A = area (m^2)
- θ = angle between \vec{m} and \vec{B}
- Maximum torque at $\theta = 90^\circ$; zero at $\theta = 0$

5.2 Magnetic Dipole Moment

$$m = N I A \quad [\text{A}\cdot\text{m}^2]$$

For circular loop: $m = N I \pi R^2$

- Direction: normal to the plane of loop (right-hand rule)

5.3 Potential Energy of a Magnetic Dipole

$$U = -\vec{m} \cdot \vec{B} = -mB \cos \theta$$

- $\theta = 0$ (aligned with B): $U = -mB$ (stable equilibrium)
- $\theta = 180$ (anti-aligned): $U = +mB$ (unstable)
- $\theta = 90$: $U = 0$ (reference position)

Work done to rotate from θ_1 to θ_2 :

$$W = mB(\cos \theta_1 - \cos \theta_2)$$

Analogy:

Magnetic dipole in $\vec{B} \longleftrightarrow$ Electric dipole in \vec{E}
 $\tau = mB \sin \theta \longleftrightarrow \tau = pE \sin \theta$
 $U = -mB \cos \theta \longleftrightarrow U = -pE \cos \theta$

6. GALVANOMETER, AMMETER & VOLTMETER

6.1 Galvanometer (Moving Coil)

$$\tau_{\text{coil}} = N I A B = \kappa \phi$$

At equilibrium: $\phi = \frac{N I A B}{\kappa}$

Current sensitivity: $S_I = \frac{\phi}{I} = \frac{N A B}{\kappa}$

Voltage sensitivity: $S_V = \frac{S_I}{R_G}$

- κ = restoring couple per unit twist (torsion constant)
- ϕ = deflection angle (rad)
- R_G = galvanometer resistance (Ω)

6.2 Ammeter (Shunt)

$$S = \frac{I_g R_G}{I - I_g}$$

- S = shunt resistance (low, in **parallel**)
- I_g = full-scale deflection current of galvanometer
- I = maximum current to be measured
- Ideal ammeter: $R = 0$

6.3 Voltmeter (Series Resistance)

$$R_s = \frac{V}{I_g} - R_G$$

- R_s = series resistance (high, in **series**)
- V = maximum voltage to be measured
- Ideal voltmeter: $R \rightarrow \infty$

Ammeter has **low** resistance; Voltmeter has **high** resistance. Interchanging them destroys the galvanometer.

7. CYCLOTRON

Cyclotron frequency (resonance condition):

$$f_c = \frac{qB}{2\pi m}$$

Maximum KE:

$$K_{\max} = \frac{q^2 B^2 R_{\max}^2}{2m}$$

Maximum speed:

$$v_{\max} = \frac{qBR_{\max}}{m}$$

- R_{\max} = radius of dee (m)
- Cyclotron **cannot** accelerate electrons (they become relativistic quickly)
- Cyclotron **cannot** accelerate neutral particles
- Frequency of oscillating electric field must equal f_c

Cyclotron frequency is **independent** of speed and radius — this is the key principle. If v increases, r increases proportionally so $T = 2\pi r/v$ stays constant.

8. QUICK COMPARISON: ELECTRIC vs MAGNETIC QUANTITIES

Quantity	Electric	Magnetic
Fundamental Law	Coulomb's Law	Biot-Savart Law
Field Law	Gauss's Law: $\oint \vec{E} \cdot d\vec{A} = q/\epsilon_0$	Ampere's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$
Dipole Moment	$\vec{p} = q\vec{d}$	$\vec{m} = NI\vec{A}$
Torque	$\tau = pE \sin \theta$	$\tau = mB \sin \theta$
Potential Energy	$U = -pE \cos \theta$	$U = -mB \cos \theta$
Work done on charge	$W = qEd$ (can do work)	$W = 0$ (magnetic force does no work)
Field of infinite plane	$E = \sigma/2\epsilon_0$	$B = \mu_0 K/2$

9. TOP 10 MUST-REMEMBER FORMULAS FOR NEET

1. Lorentz Force: $F = qvB \sin \theta$ [most fundamental]
2. Radius of circular motion: $r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$
3. Time period in B field: $T = \frac{2\pi m}{qB}$ [independent of v]
4. Force on current wire: $F = BIL \sin \theta$
5. Force per unit length between wires: $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$
6. Field at centre of circular loop: $B = \frac{\mu_0 NI}{2R}$
7. Field inside solenoid: $B = \mu_0 nI$
8. Torque on current loop: $\tau = NIAB \sin \theta = mB \sin \theta$
9. Shunt for Ammeter: $S = \frac{I_g R_G}{I - I_g}$
10. Velocity selector: $v = \frac{E}{B}$

10. LAST-MINUTE REVISION TIPS

Direction Rules — Memorise These:

- **Right-hand thumb rule:** Thumb \rightarrow current, curled fingers \rightarrow field direction (for straight wire)
- **Right-hand palm rule:** For circular loop, curl fingers in current direction, thumb $\rightarrow \vec{B}$
- **Fleming's Left Hand rule:** For force on current/charge: Fore $\rightarrow \vec{B}$, Middle $\rightarrow I$ (or v), Thumb \rightarrow Force

Top Traps in NEET MCQs:

- Magnetic force does **no work** \rightarrow speed unchanged
- $T = 2\pi m/qB$ is **independent of speed**
- Solenoid uses n (turns/m), **not** N (total turns)
- Same-direction currents **attract**, not repel
- $r \propto \sqrt{m}$ at same KE; $r \propto m$ at same v
- Galvanometer \rightarrow Ammeter: add **low** shunt in **parallel**
- Galvanometer \rightarrow Voltmeter: add **high** resistance in **series**

Quick Mental Checks:

- If $\vec{v} \parallel \vec{B}$: **no force**, straight path
- If $\vec{v} \perp \vec{B}$: **circular** path
- If \vec{v} at angle to \vec{B} : **helical** path
- $B \propto 1/r$ for wire; $B \propto r$ inside solid wire
- $B = 0$ inside hollow wire; $B = 0$ outside toroid

Unit Cross-Checks:

- $[B] = \text{Tesla} = \text{kg} \cdot \text{A}^{-1} \cdot \text{s}^{-2} = \text{N}/(\text{A} \cdot \text{m})$
- $[m] = \text{A} \cdot \text{m}^2$ (magnetic moment)
- $[\mu_0] = \text{T} \cdot \text{m} \cdot \text{A}^{-1} = \text{H} \cdot \text{m}^{-1}$

Formula Pairs to Not Confuse:

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \text{ vs. } B_{\text{loop}} = \frac{\mu_0 I}{2R}$$

$$B_{\text{solenoid}} = \mu_0 n I \text{ vs. } B_{\text{toroid}} = \frac{\mu_0 N I}{2\pi r}$$

$$r = \frac{mv}{qB} \text{ vs. } r = \frac{\sqrt{2mK}}{qB}$$